
Jet and Rocket Propulsion

AE4451

LECTURE 31

- what we saw in Lecture 30
 - analysis of turbine engine components
 - inlet/diffuser (supersonic)
- today
 - midterm post-mortem

Announcements:

1. Project guidelines to be sent out this evening
2. Friday's (March 31) lecture will be given by **Prof. Sandy Magnus**, former astronaut

Midterm

1. Performance of a rocket engine

(i) find change in equivalent exhaust velocity between 6 and 9 km

• given:

- exhaust velocity
- exhaust pressure
- nozzle exit diameter
- altitudes of flight
- mass flow rate

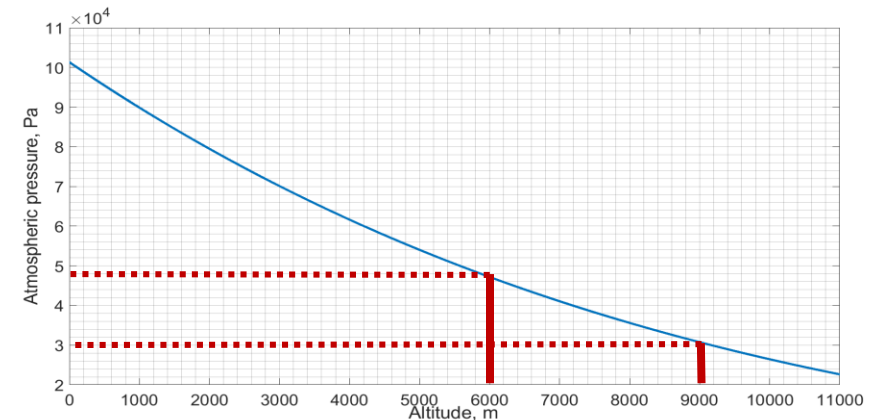
- result: [167.5 – 188.5 m/s]

• grading notes:

- incorrect area, otherwise correct: 5/8 points given

$$u_{eq} \equiv \frac{\tau}{\dot{m}} = u_e + \frac{(p_e - p_a)A_e}{\dot{m}}$$

- find pressure at 6km, 9km, find corresponding u_{eq}
- takeaway: equivalent velocity affected by altitude



(ii) relationship between stagnation pressure and throat area for thrust coefficient > 1

$$c_\tau \equiv \frac{\tau}{A_t p_o}$$

- at 9km: know u_{eq} , flow rate \rightarrow calculate thrust

- result: $A_t p_o < 11270$

• grading notes:

- incorrect inequality, otherwise correct: 4/6 points

1. Performance of a rocket engine

(iii) find expression relating stagnation pressure, temperature, to throat area

$$\dot{m}_{choked} = A_t \frac{P_o}{\sqrt{RT_o}} \sqrt{\gamma \left(\frac{2}{\gamma+1} \right)^{\frac{\gamma+1}{\gamma-1}}}$$

- now given:
 - wording indicating that choked flow condition is met
 - γ value

- result: $\frac{A_t P_o}{\sqrt{RT_o}} = 4.559$

- grading notes:
 - incorrect substitution, e.g. γ value = 5/6 points
 - incorrect or incomplete evaluation = 3/6 points

(iv) what constraints are there on f , fuel-oxidizer ratio?

- most important: temperature (commonly use incomplete combustion to lower chamber temperature, fuel-rich, not stoichiometric)
- cannot use any value of f in tuning thermal efficiency
 - grading notes:
 - full points for similar answer (recognizing thermal, material constraint)

Midterm

2. Control volume analysis

(i) write mass conservation expression

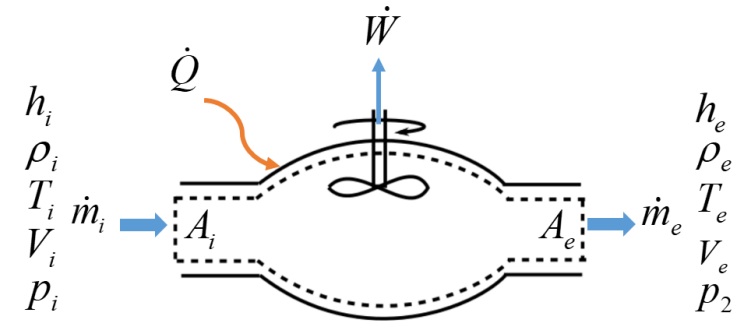
- given:
 - mass in control volume increasing at rate a

- result:

$$a = \dot{m}_i - \dot{m}_e$$

$$a = \rho V_i A_i - \rho V_e A_e$$

- grading notes:
 - either expression acceptable
 - incorrect sign = 1 out of 2 points



(iv) show RTT allows us to write

$$\dot{Q} - \dot{W} = \dot{m}_e \left(h_e + \frac{V_e^2}{2} \right) - \dot{m}_i \left(h_i + \frac{V_i^2}{2} \right)$$

use
$$\dot{Q}_{in} - \dot{W}_{shaft} = \frac{d}{dt} \int_{CV} \rho e_o dV + \int_{CS} \rho h_o (\vec{u} \cdot \hat{n}) dA$$

- grading notes:
 - straightforward problem; all points awarded

Midterm

3. Propulsion system thermodynamics

- given following reaction: $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$

please keep in mind for future problems

(i) write equilibrium constant in terms of partial pressures

use
$$\frac{\prod_{j,RHS} (p_j / p^o)^{\nu_j}}{\prod_{i,LHS} (p_i / p^o)^{\nu_i}} = K_p(T)$$
 result:
$$K_p = \frac{(p_{NO} / p_{ref})^2}{(p_{O_2} / p_{ref})(p_{N_2} / p_{ref})}$$

- grading notes:
 - square brackets normally reserved for concentrations, not partial pressures (not penalized)
 - correct formula, but incorrect application of exponents: 1/3 points

(ii) what does K_p indicate qualitatively about balance of reactants, products?

$K_p \sim 10^{-11}$

- reactant partial pressure greatly exceeds product partial pressure
- mostly reactants present

$K_p \sim 1$

- reactant and product partial pressures on same order
- similar amounts of reactants, products present

note that K_p isn't used to determine whether reaction is *balanced* or not, significance of K_p is that different T s will set up different equilibria

- grading notes:
 - understanding of balance: full points

Midterm

3. Propulsion system thermodynamics

- given following reaction: $N_{2(g)} + O_{2(g)} \rightarrow 2NO_{(g)}$

	P_{N_2}	P_{O_2}	P_{NO}
Initial mixture	0.78	0.21	0
Final mixture	$0.78 - x$	$0.21 - x$?

(iii) find partial pressure of NO in final mixture

- recognize mass not created
- total pressure of all species should add up to same value in initial and final states
- alternatively: N_2 decreases by x , O_2 decreases by x , can only go to NO ($2x$)

result: $2x$

(iv) given K_p , determine x

- use expression from (i)
$$K_p = \frac{(p_{NO} / p_{ref})^2}{(p_{O_2} / p_{ref})(p_{N_2} / p_{ref})}$$

$$2 \times 10^{-31} = \frac{(2x)^2}{(0.78 - x)(0.21 - x)}$$

- grading notes:
 - $x = 0$: this value also accepted
 - incorrect formula, substitutions, resolution, but recognition of equation to solve: 2/7 points
 - correct formula, incorrect substitutions, attempt at solving: 3/7 points
 - correct formula, just missing final calculation: 5/7 points

result: $x \sim 10^{-17}$: tiny value; for these conditions, mostly reactants present

3. Propulsion system thermodynamics

- given new reaction: $N_{2(g)} + 2O_{2(g)} \rightarrow NO_{(g)} + NO_{3(s)}$

(v) write Kp

- recognize that partial pressure of solid = 1 (will not appear in final expression for Kp)

result:
$$K_p = \frac{(p_{NO} / p_{ref})}{(p_{N_2} / p_{ref})(p_{O_2} / p_{ref})^2}$$

- grading notes:
 - correct expression with exponents, not recognizing solid state: 1.5/3 points
 - incorrect use of exponents, recognition of solid state: 1.5/3 points

Observations

- overall:
 - calculations reasonable; necessary to know what formula to use on equation sheet
: a good understanding of the underlying material makes this easier
 - partial credit awarded where reasoning could be followed, even with incorrect/unfinished calculations
 - on "simple" questions, e.g. mass flow rate: more precision necessary